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JV-003-001513 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October – 2019

Mathematics : Paper - BSMT - 501(A)

(Mathematical Analysis & Group Theory)

Faculty Code : 003

Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 Answer the following questions briefly : **20**

(1) If $E = [-1, 1]$ is subset of metric space R then find E^0.

(2) Define lower Riemann Integration.

(3) Define Open Set.

(4) Define Derived Set.

(5) If (X, d) is a discrete metric space then $N(a.1/2) =$ ....

(6) Define Neighborhood.

(7) Define Norm of Partition.

(8) For group $\left(z_5, +_5\right)$, then $o(4) =$ ....

(9) Write the order of set of fourth roots of 1.

(10) Write the number of element in symmetric group S_3 is

(11) Define Normal Subgroup.

(12) Define Quotient Group.

(13) In usual notation $S(P, f)$ = ...

(14) Find the order of the cyclic subgroup generated by element 2 of Z_8.

(15) Write the number of even permutation in symmetric group S_3 is

(16) If $f:[1, 3] \to R, \ f(x) = \frac{1}{x}$ and $P = \{1, 2, 3\}$ then $U(P, f)$ = ...

(17) For group $(C, \times)$, find the inverse element of i^{121}.

(18) Define Finer Partition.

(19) If $\mu = (1\ 5)\ (3\ 4) \in S_6$ then $o(\mu)$ = ...

(20) Every subgroup of commutative group is normal. (say True or False)

2 (a) Answer any three : **6**

(1) If f is continuous bounded function over $[a, b]$ then for any two partitions P_1 and P_2 prove that $L(P_1, f) \leq U(P_2, f)$.

(2) If (X, d) is a metric space and $A, B \subset X$ and $A \subset B$ then $A' \subset B'$.

(3) Obtain border set of the subset (1, 2) of metric space R.

(4) Determine whether set $\{x \in R / x^2 - 2x - 1 = 0\}$ is open or closed set.

(5) Prove that every finite subset of any metric space is closed set.

(6) Evaluate : $\lim\limits_{n \to \infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + ... + \left(\frac{n}{n}\right)^2\right]$.

(b) Answer any three : **9**

(1) State and prove first mean value theorem of R-integration.

(2) If f is decreasing function in $[a, b]$ then prove that f is R-integrable.

(3) If function f is increasing in $[a, b]$ then prove that f is Riemann integrable on $[a, b]$.

(4) State and prove principle of Housedorff's in metric space.

(5) If (X, d) is a metric space and $A, B \subset X$ then prove

$(A \cap B)^0 = A^0 \cap B^0$.

(6) Prove that the arbitrary union of open sets of metric space is an open set.

(c) Answer any two : **10**

(1) Prove that

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\ldots\ldots\ldots+\frac{1}{8n}\right]=\frac{3}{8}.$$

(2) State and prove necessary and sufficient condition for a bounded function to be R-integrable.

(3) Prove that $\frac{\pi^2}{6} \le \int_0^{\pi} \frac{x}{2+\cos x}\,dx \le \frac{\pi^2}{2}$.

(4) Prove that $\frac{1}{4}$ is in cantor set.

(5) Prove that Closer set of any subset of a metric space is a closed set.

3 (a) Answer any three : **6**

(1) Prove that the identity element is unique in the group.

(2) Give the example that the union of two subgroups of a group may not be a subgroup.

(3) If G is a group then show that the set of all elements satisfies the equation $x^2 = e$ is a subgroup of G.

(4) Check whether $(Z, +)$ is cyclic group or not.

(5) If $\sigma = (1\ 8)(3\ 6\ 4)(5\ 7),\ \sigma \in S_8$ then determine whether σ is even or odd.

(6) Define : Isomorphism between groups.

(b) Answer any three : **9**

(1) In a finite group G, prove that $o(a) / o(G)$ for each $a \in G$.

(2) Prove that a group of prime order is cyclic.

(3) Draw the lattice diagram of S_3.

(4) Show that the relation $a \equiv b(\text{mod } H)$ is an equivalence relation in group G where $H \leq G$.

(5) If K is a subgroup of G and H is a normal subgroups of group G then prove that $H \cap K$ is also a normal subgroup of K.

(6) Prove that intersection of two subgroups of a group is again a subgroup.

(c) Answer any two : **10**

(1) State and prove Cayley's theorem.

(2) Prove that the combination of two disjoint cycles in S_n is commutative.

(3) Prove that in symmetric group S_n, the number of elements in the set of even permutation A_n is equal to the number of elements in the set of odd permutation B_n and it is $\frac{n!}{2}$, $n \geq 2$.

(4) State and prove Lagrange's theorem for finite groups.

(5) A subgroup H of a group G is a normal subgroup of $G \Leftrightarrow aHa^{-1} \subset H;\ \forall\ a \in G$.